Apple Bread Challenge[2]

Challenge 2:
Due Date: March 25th, 2024 (via email)

We shall simulate and understand the One-Dimensional Annihilation-Creation, $\{A_n\}_{n\geq 1}$, from [EW99]. We describe the model below:

(a) Let $A_0 = \{0\}$(i.e. 0 is occupied) and fix some integer $K \geq 2$.

(b) Given A_n, (i.e. finite sites occupied by at time n) the set A_{n+1} is constructed by first adding K sites one by one and then deleting $K-1$ sites one by one. More precisely,

(i) To add a particle, its new site is chosen uniformly from the boundary of the given set of occupied sites, (a site y belongs to the boundary ∂a of a set $a \subset \mathbb{Z}$ if $y \in \mathbb{Z} \setminus a$ and if there exists $x \in a$ with $|x-y| = 1$).

(ii) To delete a particle, a site is chosen uniformly from the set of all occupied sites.

1. How much does A_n grow at each step ?
2. Can you write an `R` that simulates the process A_n at each time n ? Using `gganimate` (or otherwise), can you produce a dynamic simulation of the process over n for say 1000 steps with K=3 ?
3. A gap of $A \subset \mathbb{Z}$ is defined as a nonempty subset g of $\mathbb{Z} \setminus A$ with the property that there exist $x_1, x_2 \in A$ with $g = \{y \in \mathbb{Z} : x_1 < y < x_2\}$). Let $G(A)$ denote the number of gaps of A, let $G^{(2)}(A)$ denote the number of gaps with $|g| \geq 2$ and let $L(A)$ denote the number of unoccupied sites between the endpoints of A, i.e., $L(A) = |g_1| + \ldots + |g_{G(A)}|$, where $g_1, \ldots, g_{G(A)}$ are the different gaps of A.

(Step 1) For each $n = 1, \ldots 1000$, simulate A_n and compute $L(A_n), G^{(2)}(A_n)$

(Step 2) Replicate Step 1, 100 times and set L_n be the average of $L(A_n)$ over the 100 trials for each n and $G_n^{(2)}$ be the average of $G^{(2)}(A_n)$ over the 100 trials for each n.

(Step 3) Decide if it is likely that $L_n = O(h_n)$ where $h(n) = \log n / \log\log n$ (use idea from (4) in Book-keeping exercise)

(Step 4) Plot $G_n^{(2)}$.

Check the results [EW99, Theorem 2.1.1 and Theorem 2.1.2, Pages7-8].

References

[EW99] Dorothea Maria Eberz-Wagner. *Discrete growth models.* Thesis: University of Washington, https://arxiv.org/pdf/math/9908030.pdf, 1999.

[2]This will be a Three-part challenge each with a specific due date. If I get a complete solution to the three-part challenge [From 5 students] by due dates then I will bake apple bread for everyone in class who has 90% attendance.