ISCD : Challenge 1

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```r
library(viridis)

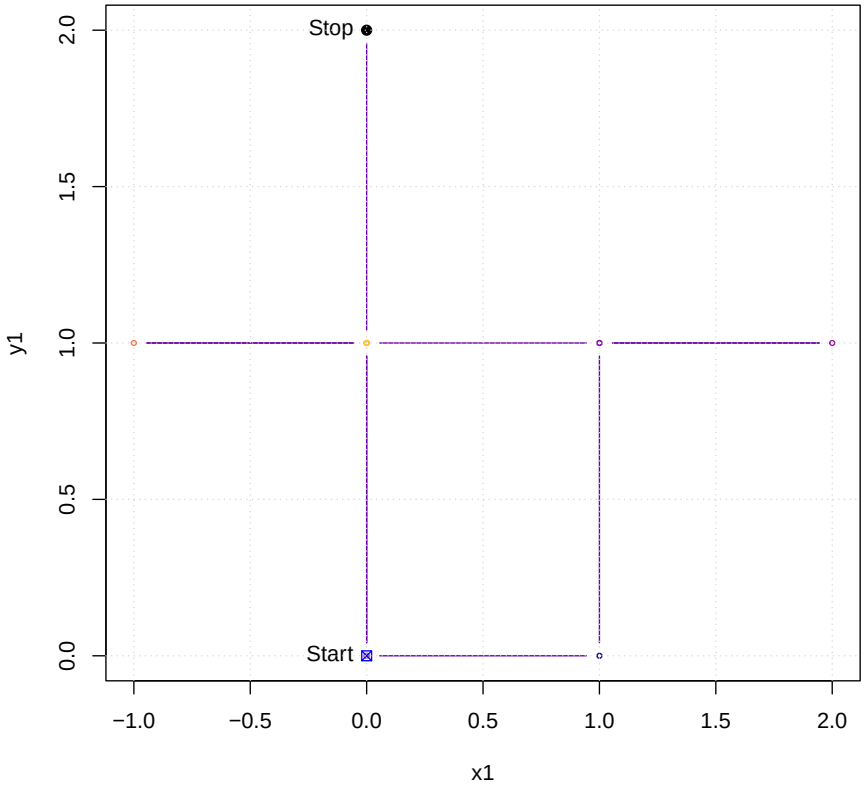
## Loading required package:  viridisLite
```

Challenge Problem 1 :

(a)

Theoretically, the range includes all lattice points given by the equation $\mathrm{R} = \{(x, y) \in \mathbb{Z}^2 : |x| + |y| \leq \ 10, \ x + y \in \ 2\mathbb{Z}\}$.

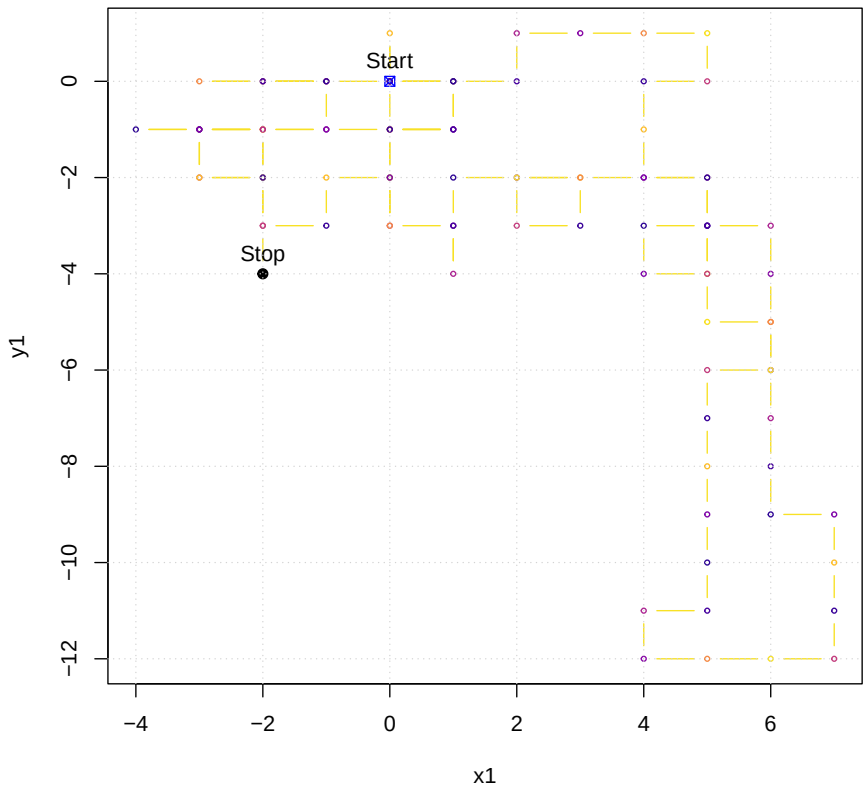
```r
x1=c(0);y1=c(0);#starts at origin
for(j in 1:10){
  s=sample(c(1,2,3,4),size=1)#simulates random walk on Z^2
  if(s==1)#moves right
    x1[j+1]=x1[j]+1
  if(s==2)#moves left
    x1[j+1]=x1[j]-1
  if(s==3)#moves up
    y1[j+1]=y1[j]+1
  if(s==4)#moves down
    y1[j+1]=y1[j]-1
  if(length(x1)>length(y1))
    y1[j+1]=y1[j]
  if(length(x1)<length(y1))
    x1[j+1]=x1[j]
}
plot(x1,y1,col=sample(plasma(37),10),type="b",cex=0.5)
#Plots paths of random walk
points(c(0),c(0),pch=7,col="blue")
points(c(x1[length(x1)]),c(y1[length(y1)]),pch=19,col="black")
text(c(0,x1[length(x1)]),c(0,y1[length(y1)]),labels=c("Start","Stop"), pos = 2)
grid()
```


Stop
Start
y1
x1
2.0
1.5
1.0
0.5
0.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

(b)

Theoretically, the range includes lattice points given by the equation $\mathrm{R} = \{(x, y) \in \mathbb{Z}^2 : |x| + |y| \leq \ 100, \ x + y \in \ 2\mathbb{Z}\}$.

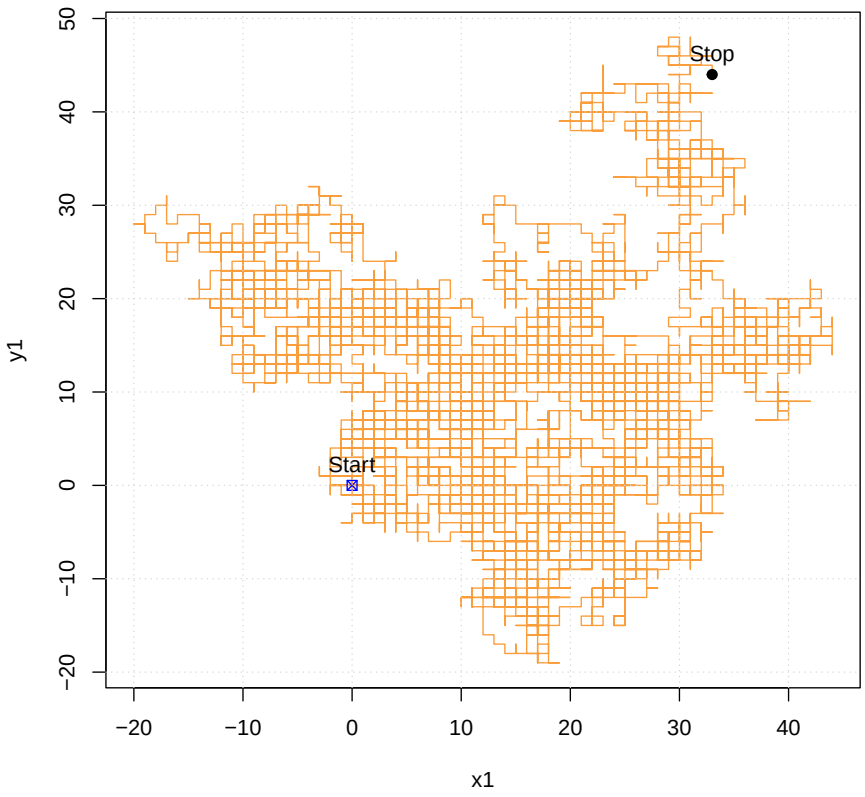
```
x1=c(0);y1=c(0);
for(j in 1:100){
  s=sample(c(1,2,3,4),size=1)
  if(s==1)#moves right
    x1[j+1]=x1[j]+1
  if(s==2)#moves left
    x1[j+1]=x1[j]-1
  if(s==3)#moves up
    y1[j+1]=y1[j]+1
  if(s==4)#moves down
    y1[j+1]=y1[j]-1
  if(length(x1)>length(y1))
    y1[j+1]=y1[j]
  if(length(x1)<length(y1))
    x1[j+1]=x1[j]
}
plot(x1,y1,col=sample(plasma(37),10),type="b",cex=0.5)
points(c(0),c(0),pch=7,col="blue")
points(c(x1[length(x1)]),c(y1[length(y1)]),pch=19,col="black")
text(c(0,x1[length(x1)]),c(0,y1[length(y1)]),labels=c("Start","Stop"), pos = 3)
grid()
```


Start
Stop
y1
x1
0
−2
−4
−6
−8
−10
−12
−4
−2
0
2
4
6

(c)

The only points allowed are ($\pm$ 55,0) and (0,$\pm$ 55). Since there's no limit to the number of steps allowed, the range is the entire $\mathbb{Z}^2$ plane.

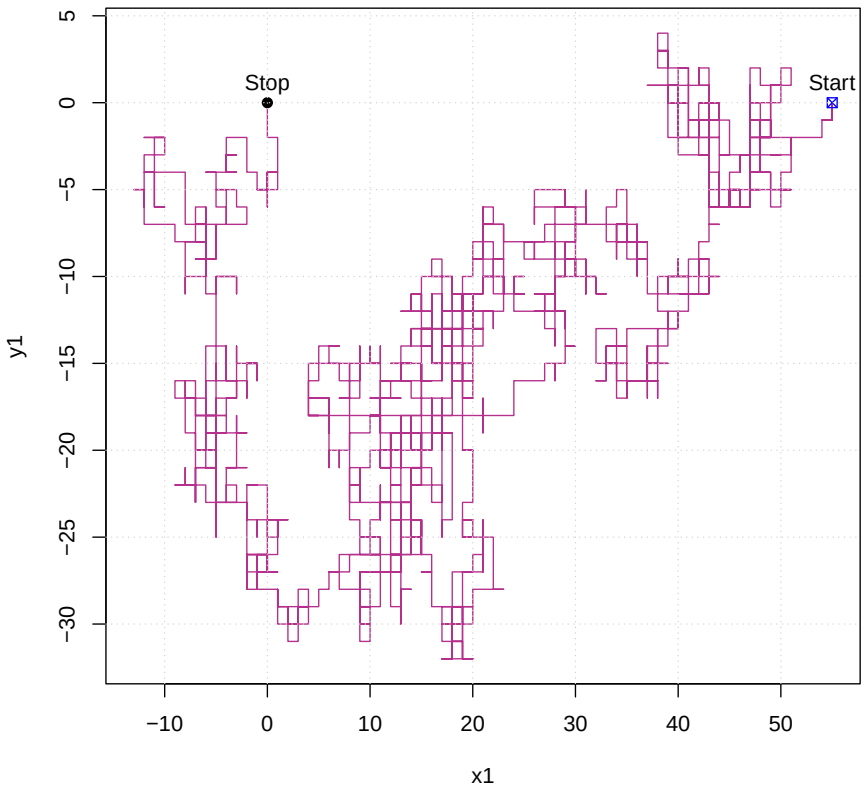
```
x1=c(0);y1=c(0);#starts at (0,0)
j=1
while(x1[j]^2+y1[j]^2!=55^2){#till distance from (0,0)!=55
  s=sample(c(1,2,3,4),size=1)
  if(s==1)#moves right
    x1[j+1]=x1[j]+1
  if(s==2)#moves left
    x1[j+1]=x1[j]-1
  if(s==3)#moves up
    y1[j+1]=y1[j]+1
  if(s==4)#moves down
    y1[j+1]=y1[j]-1
  if(length(x1)>length(y1))
    y1[j+1]=y1[j]
  if(length(x1)<length(y1))
    x1[j+1]=x1[j]
    j=j+1
}
plot(x1,y1,col=sample(plasma(37),10),type="l")
points(c(0),c(0),pch=7,col="blue")
points(c(x1[length(x1)]),c(y1[length(y1)]),pch=19,col="black")
text(c(0,x1[length(x1)]),c(0,y1[length(y1)]),labels=c("Start","Stop"), pos = 3)
grid()
```


Stop
Start
y1
x1
50
40
30
20
10
0
−10
−20
−20
−10
0
10
20
30
40

(d)

The only point allowed is (0,0). Since there's no limit to the number of steps allowed, the range is the entire $\mathbb{Z}^2$ plane.

```
x1=c(55);y1=c(0);#starts at (55,0)
j=1
while(x1[j]!=0 || y1[j]!=0){#till walk doesn't hit (0,0)
  s=sample(c(1,2,3,4),size=1)
  if(s==1)#moves right
    x1[j+1]=x1[j]+1
  if(s==2)#moves left
    x1[j+1]=x1[j]-1
  if(s==3)#moves up
    y1[j+1]=y1[j]+1
  if(s==4)#moves down
    y1[j+1]=y1[j]-1
  if(length(x1)>length(y1))
    y1[j+1]=y1[j]
  if(length(x1)<length(y1))
    x1[j+1]=x1[j]
    j=j+1
}
plot(x1,y1,col=sample(plasma(37),10),type="l")
points(c(55),c(0),pch=7,col="blue")
points(c(x1[length(x1)]),c(y1[length(y1)]),pch=19,col="black")
text(c(55,x1[length(x1)]),c(0,y1[length(y1)]),labels=c("Start","Stop"), pos = 3)
grid()
```


Stop
Start
y1
x1
5
0
−5
−10
−15
−20
−25
−30
−10
0
10
20
30
40
50