Due: March 29th, 2024,10pm
Problems Due: 1

1. Below we provide a conjecture. Post deciding on a suitable parameter about which the conjecture is about frame the null and alternate hypothesis for each. For e.g.

   Conjecture : *The mean retirement age in India is 65.*

   Let μ be mean retirement age. *Null Hypothesis:* $\mu = 65$ and *Alternative Hypothesis:* $\mu \neq 65$

   (a) The mean household income in India is Rs 50000.

   (b) Siva's mean commute time to ISI is atleast 90 min.

   (c) ISCD class members mean delay arrival time for 10am class is at most 15min.

   (d) The chance that superstudent will submit homework late is under 15%

2. Suppose $X_1, X_2, X_3, \ldots X_{100}$ are sampled from a population having Normal distribution with mean μ and variance 3. It turns out that $\bar{X} = 10.2$

   (a) Can you decide the validity in anyway of the conjecture that $\mu = 8.5$?

   (b) Can you decide the validity in anyway of the conjecture that $\mu = 9.5$?

3. Siva believes that fewer than 15% of people in his class listen to anything he says. In a randomly chosen sample of 960 students 380 said they listen to Siva. Frame a conjecture as the null hypothesis and an approprite alternative hypothesis. Under normality assumption with known variance of 2 decide with what confidence will(if at all) be able to reject the null hypothesis.

Book-Keeping Exercises[1]

1. Suppose we toss a fair coin two times. Consider the three events $A_1 = \{hh, tt\}$, $A_2 = \{hh, ht\}$, and $A_3 = \{hh, th\}$. Compute

$$P(A_1), P(A_2), P(A_3), P(A_1 \cap A_2), P(A_2 \cap A_3), P(A_1 \cap A_3), \text{ and } P(A_1 \cap A_2 \cap A_3).$$

   What can you conclude ?

2. A pair of random variables (X, Y) is called "bivariate normal" if $aX + bY$ is a normally distributed random variable for all real numbers a and b.

   (a) Suppose (X, Y) and (Z, W) are two bivariate normal random variables. If

$$\begin{aligned} E[X] = E[Z] = \mu_1, \qquad & E[Y] = E[W] = \mu_2 \\ Var[X] = Var[Z] = \sigma_1^2, \qquad & Var[Y] = Var[W] = \sigma_2^2 \\ \text{and} & \\ Cov[X, Y] = Cov[Z, W] = \sigma_{12} & \end{aligned} \tag{1}$$

   then show that (X, Y) and (Z, W) have the same joint distribution.

   (b) Let X_1, X_2 be two independent normal random variables with mean 0 and variance 1. Let (Y_1, Y_2) be a bivariate normal random variable with zero means, variances equal to 1 and correlation $\rho = \rho[Y_1, Y_2]$, with $\rho^2 \neq 1$. Let f be the joint probability density function of (X_1, X_2) and g be the joint probability density function of (Y_1, Y_2). For $0 < \alpha < 1$, let (Z_1, Z_2) be a bivariate random variable with joint density given by

$$h(z_1, z_2) = \alpha g(z_1, z_2) + (1 - \alpha) f(z_1, z_2),$$

   for any real numbers z_1, z_2.

   (i) Write down the exact expressions for f and g.

   (ii) Verify that h is indeed a probability density function.

   (iii) Show that Z_1 and Z_2 are Normal random variables by calculating their marginal densities.

   (iv) Show that (Z_1, Z_2) is not a bivariate normal random variable.

3. For plotting functions or calculus related matters, please refer to https://dtkaplan.github.io/RforCalculus/. Try this code and understand the graphs

```
> library(mosaicCalc)
> slice_plot(exp(-x^2/2)*(1/(2*pi)^(0.5)) ~ x, domain(x = range(-3,3)))
> library(mnormt)
> interactive_plot(
+       dmnorm(cbind(x, y), c(0,0), matrix(c(2, -1, -1, 2), nrow=2))~ y & x,
+    domain(x= -3:3, y = -3:3))
```

[1] Questions 1 and 2 are from material within the course syllabus and you may be tested on these.