Due: February 15th, 2024,10pm

1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. samples from Normal (μ, σ^2). Find the maximum likelihood estimate for μ, σ^2.

2. Let $X_1, X_2, \ldots, X_n$ be i.i.d. samples from Poisson (λ). Find the maximum likelihood estimate for λ.

3. The following code simulates 100 samples from a population with distribution Binomial $(20, 0.4)$ and computes an estimate for the sample size parameter n and the success probability p (($n = 20$ and $p = 0.4$ in the simulation).

```
> n <- 100
> N <- 20
> p <- 0.4
> x <- rbinom(n, N, p)
> m1 <- mean(x)
> m2 <- mean(x^2)
> Nhat <- m1^2 / (m1 -(m2-m1^2))
> phat <- (m1 - (m2-m1^2)) / m1
```

   (0) Describe the methodu used to compute the estimate `phat, Nhat`

   (a) Run the code in `R` and compute `phat - p` and `Nhat - N`.

   (b) Change the code suitably to simulate 1000 samples from Binomial $(20, 0.4)$ and see if the answer to (a) changes.

   (c) Change the code suitably to simulate samples from Binomial $(10, 0.1)$ and Binomial $(10, 0.9)$, and repeat (a) and (b).

4. Suppose the group learned-tossers perform 10 independent trials of tossing a biased coin with probability of heads p, 50 times. They inform us that maximum (over 10 trials) number of heads obtained was 30.

   (a) Find the likelihood (or probability) of obtaining a maximum of 30 heads in 10 independent trials of 50 tosses of biased coin with probability of heads p.

   (b) Deduce that, even though you are a powerful student at an institute of national importance, that it is next to impossible for you to find the best estimate of p using calculus in 5 minutes. Regardless provide your best possible guess.

   (c) Write a function in `R`, that will take in N, n, M, p as inputs and compute the likelihood (or probability) of obtaining a maximum of M heads in n independent trials of N tosses of biased coin with probability of heads p.

   (d) Let $N = 50, n = 10, M = 30$.

      i. Write an `R`-code that plots the the likelihood (or probability) of obtaining a maximum of M heads in n independent trials of N tosses of biased coin with probability of heads p as a function of p. From the plot provide your best estimate of p.

      ii. Using the `optimise` function find the Maximum likelihood estimate of p.