Due: March 22nd, 2024 5pm

1. (Tschebychev Inequality)

   (a) Find a random variable X with Range$(X) = \{-1, 0, 1\}$ such that

   $$P(\mid X - \mu \mid \geq 2\sigma) = \frac{1}{4},$$

   with $\mu = E[X]$ and $\sigma^2 = \text{Var}[X]$.

   (b) Construct another random variable Y (different from X) with Range $(Y) = \{y_1, y_2, y_3\}$, mean μ and with

   $$P(\mid Y - \mu \mid > 2\sigma) > P(\mid X - \mu \mid > 2\sigma),$$

   so as to get

   $$P(\mid Y - \mu \mid > 2\sigma) > \frac{1}{4}$$

   Decide whether Tschebychev Inequality is violated ?

   (c) Write an R-code that takes an input k, and constructs a random variable X with Range$(X) = \{-1, 0, 1\}$ such that

   $$P(\mid X - \mu \mid \geq k\sigma) = \frac{1}{k^2},$$

   with $\mu = E[X]$ and $\sigma^2 = \text{Var}[X]$. Further the R-code should construct a random variable Y (different from X) with Range $(Y) = \{y_1, y_2, y_3\}$, mean μ so that

   $$P(\mid Y - \mu \mid > k\sigma) > \frac{1}{k^2}$$

   and (using replications) verifies your conclusion about Tschebychev's inequality in (b). It should save the entire output as a (suitably designed) csv file.