# Find a test statistic (in general)

$X \overset{d}{=} \text{Normal}(\mu, \sigma^2)$ $\mu$ - unknown

$\sigma$ - known.

Conjecture: $\mu = a$ ie $P_0 = [a, \infty)$

level of significance $\equiv 5\%$

$X_1, X_2, \ldots, X_n$ are iid $X$ $\quad \mu \in P = \mathbb{R}$

Null: $\mu = a$ $\quad P_0 = \{a\}$

$$Z_i = \frac{X_i - \mu}{\sigma} \; ; \; Z_i \overset{d}{=} \text{Normal}(0,1)$$

Statistic: $\sqrt{n}\left(\frac{\bar{X} - \mu}{\sigma}\right) \overset{d}{=} \text{Normal}(0,1$

Under the Null : If (Null hypothesis is true) $\mu = a$ then

$$T_n(\bar{X}) = \sqrt{n}\left(\frac{\bar{X} - a}{\sigma}\right) \xrightarrow{d} \text{Normal}(0,1)$$

Suppose $Y_1, Y_2, \ldots Y_n$ are i.i.d Normal$(a, \sigma^2)$

then $T_n(\bar{Y}) = \sqrt{n}\left(\frac{\bar{Y} - a}{\sigma}\right) \xrightarrow{d}$ Normal$(0,1)$

observed data $X_1, \ldots, X_n$ depends on unknown $\mu$

& hypothetical $Y_1, \ldots Y_n$ random variab

are distributionally the same under the

## Null hypothesis

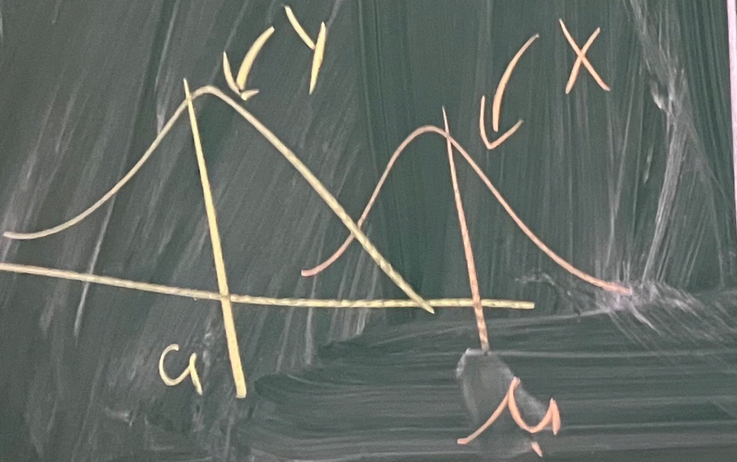
To test the null hypothesis $\mu = a$ vs

or not

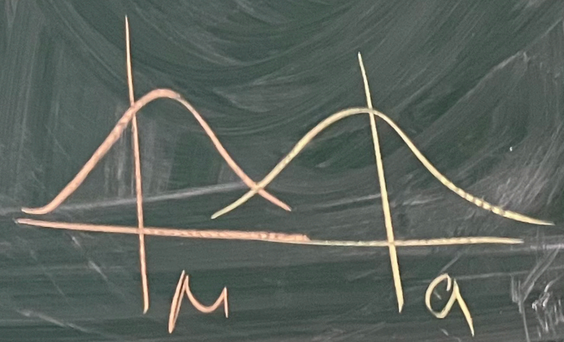
We will check the value of $P(T(\bar{Y}) > T(\bar{X}))$

- Versus
  - $\mu \neq a$
  - $\mu > a$
  - $\mu < a$

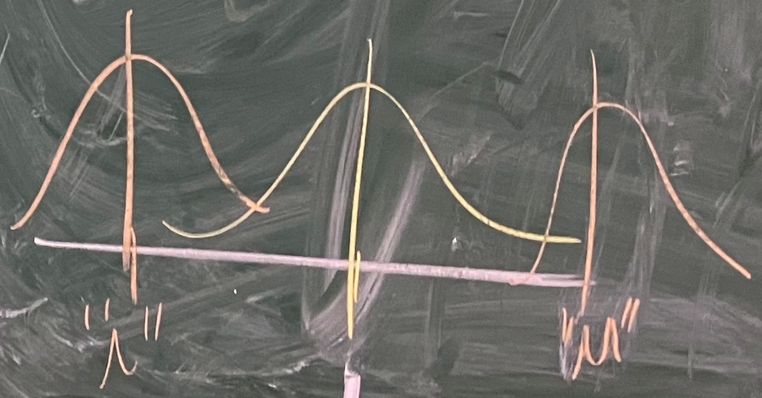
$\mu > a$




$\mu < a$




$\mu \neq a$



Null:- $\mu = a$

Alternate: $\mu \neq a$

$\mathbb{P}(|T(\bar{Y})| > |T(\bar{x})|)$

$< 0.05$

then there is evidence to reject the null

$\mu = a$

$\mu > a$

If

$\mathbb{P}(T(\bar{Y}) > T(\bar{x}))$

$< 0.05$

then there is evidence to reject Null!

$\mu = a$

$\mu < a$

If

$\mathbb{P}(T(\bar{Y}) < T(\bar{x}))$

$< 0.05$

then there is evidence to reject null