**Due: February 9th, 2024, 12pm**
*Problem due: 5*

1. Suppose $X$ is the number of heads of a biased coin with probability of heads $p$ in $N$ tosses. Then find the distribution of $X$.

2. Suppose we toss a biased coin with probability of heads $p$, 45 times and the number of heads observed is 26.

   (a) Find your best estimate of $p$.

   (b) Write down the likelihood (or probability) of observing 26 heads in 45 tosses of a biased coin with probability of heads $p$, denote it by $L(p)$.

   (c) Using Calculus find the $\hat{p} = \arg\max\{L(p) : p \in [0,1]\}$

   (d) After implementing in `R` describe in two sentences what is the code doing

```
> N=45
> x=26
> p = 0.5
> choose(N,x) * p^x*(1-p)^(N-x)
```

3. After implementing in `R`

```
> N=45
> x=26
> seqp= seq(0,1, by = 1/100)
> Lseqp = choose(N,x) * seqp^x*(1-seqp)^(N-x)
> plot(x = seqp, y = Lseqp, type="l" )
```

   describe what is being accomplished by doing `seqp`,`Lseqp`.

4. After implementing the below `R`

```
> L = function(p){
+   N=45
+   x=26
+   choose(N,x) * p^x*(1-p)^(N-x)
+ }
> plot(L, from=0, to =1)
> optimize(L, interval=c(0,1), maximum=TRUE)
```

   Describe what is being accomplished by the code

5. Suppose we toss a coin repeatedly and observe that the first head appears in the 5-th trial.

   (a) Find the m.l.e. for the probability of heads when one tosses the coin.

   (b) Using the `function` command, write an `R`-code that will take in $N$ as input, find the likelihood and the m.l.e. for the probability of heads when one observes that the first head at the $N$-th trial.

**Due: February 9th, 2024, 1pm**

**Construction of Erdös Renyi Graph with** 10 **vertices**

1. Tear five small pieces of paper, and write down numbers $1, 2, 3, 4, 5$. Fold each paper so that the number written on it is not visible. Shuffle the chits, and choose one chit at random. Let $x$ denote the chosen number. Remember this $x$.
2. Consider the experiment of rolling a die and specify an event from that experiment which occurs with probability $x/6$. Let that event be called $C$. Write out the description of the event $C$.
3. The set of vertices of the graph are $\{1, 2, \ldots, 10\}$. What is the total number of possible edges, not including self-edges ?
4. Construct the *random* adjacency matrix $A$ for the graph as follows. For each pair $1 \leq i < j \leq 10$:
   (a) Roll your die and observe if the event $C$ has occured.
   (b) Designate
   $$a_{ij} = \begin{cases} 1 & \text{if } C \text{ occured.} \\ 0 & \text{if } C \text{ did not occur} \end{cases}$$
   Complete the matrix by using the properties of Adjacency matrix of undirected graphs with no self-loops.
5. Draw the random graph, denote by $G(10, \frac{x}{6})$, corresponding to the above adjacency matrix[1]

[1]Note that if you were to repeat this whole experiment again, you would obtain a completely different graph. This is why this is called a *random graph*.

**Due: February 9th, 2024, 5:00pm**

Let $x$, $A$, $C$, be as in the previous worksheet and also let $G(10, \frac{x}{6})$ be the graph constructed.

1. Fill in the following table:

| $x$ | $G(10, \frac{x}{6})$ | # Edges $E$ | # Triangles $T$ | # Squares $S$ |
|---|---|---|---|---|
| | | | | |

2. Fix any $x \in \{1, 2, 3, 4, 5\}$. Let $E$ be the number of edges in $G(10, \frac{x}{6})$.
   (a) Find the Range of the random variable $E$.
   (b) Find the probability of the event $\{E = e\}$ and denote it by $f(x \mid e)$.
3. Given $e \in \text{Range}(E)$, pretend $x \in [1, 5]$ and using calculus find $x^*$ that maximizes $f(x \mid e)$ with respect to $x$.
4. Substitute your value of $e$ (number of edges) from the table into the expression for $x^*$. *The resulting $x^*$ should be close to your chosen $x$ in the previous worksheet.*
5. Using the `igraph` package write an `R`-codethat will draw the graph constructed in the previous worksheet. *You may use the below code and modify suitably.*

```
> library(igraph)
> # Create a graph with 5 vertices and edges between 1,2; 1,3; 2,4; and 4,5
> graph <- graph(edges=c(1,2, 1,3, 2,4, 4,5), n=5, directed=FALSE)
> # Plot the graph
> plot(graph, layout=layout.circle, vertex.label=1:3,
+      vertex.size=50,
+      main="Graph with 3 Vertices and Edges (1-2, 1-3, 2-3)")
```