Due: February 16th, 2024, 1pm

Working with Normal Random Variables

1. Write an R-code to simulate a sample from X when has a

   (a) density given by

   $$f(x) = \begin{cases} 2.5x^{\frac{3}{2}} & 0 < x < 1 \\ 0 & \text{Otherwise.} \end{cases}$$

   (b) distribution function given by

   $$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2+x}{2} & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

2. Let X be a Normal Random variable with mean $\mu = 3$ and standard deviation $\sigma = 1$.

   (a) Using the Normal tables provided in page 2 compute the $P(2.1 < X < 3.4)$

   (b) Use the pnorm function in R to compute the $P(2.1 < X < 3.4)$

3. Let X be a Normal Random variable with mean $\mu = 3$ and standard deviation $\sigma = 4$. Compute

   (a) Compute $P(\mid X - \mu \mid < \sigma)$

   (b) Compute $P(\mid X - \mu \mid < 2\sigma)$

   (c) Compute $P(\mid X - \mu \mid < 3\sigma)$

   by two methods: using the Normal tables provided in page 2 and pnorm command.

Normal Tables

$\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp(-\frac{y^2}{2})$

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	0.5398
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	0.5793
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	0.6179
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	0.6554
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	0.6915
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	0.7257
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	0.7580
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	0.7881
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	0.8159
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	0.8413
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	0.8643
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	0.8849
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	0.9032
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	0.9192
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9332
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	0.9452
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	0.9554
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	0.9641
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	0.9713
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	0.9772
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0.9821
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0.9861
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0.9893
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0.9918
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0.9938
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0.9953
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9987
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998

Due: February 16th, 2024, 5:00pm

1. Rolling a die.

```
> x = c(1,2,3,4,5,6)
> probx= c(1/4,1/8,1/8,1/8,1/8,1/8,1/4)
> F16=sample(x, size=1500,  replace=T, prob=probx)
```

   (a) Describe what each `R` command is performing in the above.

   (b) Using the `mean` and `var` command find the mean and variance of `F16`. From this information alone what would you conclude is the range of the random variable `F16`.

   (c) Does the mean and variance from the sample generated compare closely with the true mean and variance of `F16`.

2. Tossing a coin 10 times.

```
> b1 = rbinom(100,10,0.5)
> b2 = rbinom(100,10,0.25)
> b3 = rbinom(100,10,0.75)
```

   (a) Using the `?rbinom` explain what each of the above commands is performing in `R`

   (b) Using the `mean` and `var` command find the mean and variance of `b1,b2,b3`. Compare them with the true mean and variance of the Binomial distribution.

   (c) `geom_hist` command.

```
> library(ggplot2)
> df1=data.frame(b1)
> p11= ggplot(df1) + geom_histogram(mapping=aes(x=b1), color="black", fill="NA", binwidth=1)
> p21= ggplot(df1) +
+         geom_histogram(mapping=aes(x=b1, y=..density..), color="black", fill="NA", binwidth=1)
```

      i. Explain what are the plots p11,p21 providing.

      ii. Rewrite the code to provide the plots for `b2` and `b3`.

      iii. What can you say about the three plots ?

   (d) Density Approximation. The below code plots the function `density` fin the interval $(0, 10)$ with $a = 5, s = \sqrt{2.5}$ along with the plot `p21`.

```
> library(ggplot2)
> density = function(x,a,s){ (1/((2*pi)^(0.5)*s ))* exp(-(x-a)^2/(2*s^2))}
> df1=data.frame(b1)
> p21= ggplot(df1) +
+          geom_histogram(mapping=aes(x=b1, y=..density..), color="black", fill="NA", binwidth=1) +
+          xlim(0,10) +
+          geom_function(fun=density, args=list(a=5,s=(2.5)^(0.5)))
```

      i. From the picture what does $\int_3^6 \texttt{density}(x, 5, \sqrt{2.5})dx$ approximate ?

      ii. If

$$\text{Area under the histogram between 3 and 7} \approx \int_a^b \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx,$$

      then what would be your guess for a and b

      iii. How would you try the same idea for `b2` and `b3` ? Would you get the same result ?