Wednesday, September 25th, 2025
Time: 2:00-5:00pm

Today's will be a worksheet class. We will learn the following topics via examples and computations:

1. Reversible chains and simulating a uniform random variable on an arbitrary (large) finite set S using Markov Chains.
2. You will need the following definition and results:
   Let X_n be a Markov Chain on S with transition matrix $\mathbf{P}$ and initial distribution μ.
   (a) A state $j \in S$ is said to be accessible from another state $i \in S$ if there is an $n \geq 0$ such that $p_{ij}^{[n]} > 0$. This is denoted by $j \leftarrow i$.
   (b) Two states i and j in S are said to communicate if i and j are each accessible from the other. This is denoted by $j \leftrightarrow i$.
   (c) A subset $C \subset S$ is said to be a communicating class if $i \leftrightarrow j$ whenever $i, j \in C$. Such a set is further called a closed communicating class if $k \not\leftarrow j$ whenever $j \in C$ and $k \in S \backslash C$.
   (d) X_n is said to be irreducible if $i \leftrightarrow j$ for all $i, j \in S$.
   (e) The period of $i \in S$ is defined as
   $$d(i) = g.c.d.\{m \geq 1 : p_{ii}^{[m]} > 0\}$$
   If $p_{ii}^{[m]} = 0$ for all $m \geq 1$, then $d(i)$ is defined to be 0. Finally, X_n is said to be aperiodic if $d(i) = 1$ for all $i \in S$.
   (a) A probability measure π on S is said to be a **stationary distribution** for the Markov chain X_n or its if
   $$\sum_{i \in S} \pi(i) p_{ij} = \pi(j).$$
   Suppose S is a finite state space and P is doubly stochastic, (i.e $\sum_{i \in S} p_{ij} = 1$ for all $j \in S$) then the uniform distribution on S is stationary. That is, if $\pi(i) = \frac{1}{|S|}$ for all $i \in S$, then π is a stationary distribution for X_n.

Instructions:

- This will be no internet, no device (phone, PC tablet, laptop etc) class. You may use notes and any hand written material.
- Each worksheet sesssion:
  - 1st 25 minutes work individually (no consulting any one),
  - second 15 minutes you may consult with anyone, and
  - final 10 minutes work individually. The Teaching assistants will announce each periods.
- Write your own solutions and do not copy solutions from anyone else. Answer each question as best as you can.
- Submit your worksheets on Time. **(5 points) for on time submission**