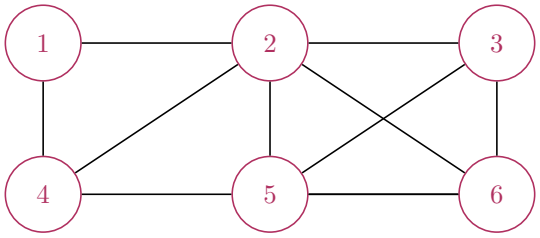
Due: Wednesday, September 25th, 2025
Time: 3:10-4:00pm

1. Let $S = \{0,1\}^m$ for $m \geq 1$. Let X_0 be any point in S. Suppose we are given X_k for $0 \leq k \leq n$ for some $n \geq 0$ and let $X_k = (s_1, s_2, s_3, \ldots, s_m) \in S$. Choose U to be a uniformly chosen number in $\{1, 2, 3, \ldots, m\}$. Define
$$X_{k+1} = (s_1, s_2, \ldots, 1 - s_U, \ldots s_m),$$
i.e. flip the U-th coordinate of X_k.
   (a) Find $\mathbf{P}$ such that X_n is a Markov chain on S with a transition matrix $\mathbf{P}$.
   (b) Find the stationary distribution π for $\{X_n\}_{n\geq 0}$.
   (c) How will you use this chain to simulate uniform random variable on S ?

2. Consider the weighted graph $G = (S, E)$ with vertex set $S = \{1, 2, 3, 4, 5, 6\}$ and edge set as given in the figure below.



   (a) Let μ denote the natural weights i.e $\mu \equiv 1$ on all edges. Let $\{X_n\}_{n\geq 0}$ be the random walk on the weighted graph (G, μ). Find the stationary distribution for $\{X_n\}_{n\geq 0}$.
   (b) Let $\{X_n\}_{n\geq 0}$ be a Markov chain on S with transition matrix $\mathbf{P}$ and initial distribution η. Suppose $\mathbf{P}$, on the vertex set S given by
$$p_{ij} = \begin{cases} \frac{1}{5} & \text{if } \{i,j\} \text{ is an edge in } G \\ 0 & \text{if } \{i,j\} \text{ is not an edge in } G, i \neq j \\ 1 - \frac{\text{degree of } i}{5} & \text{if } i = j \end{cases}$$
   Find the stationary distribution of $\{X_n\}_{n\geq 0}$
   (c) What is the significance of 5? What are the differences between the Markov chains in (a) and (b) ?

(5 points) for on time submission