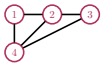
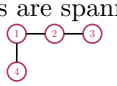
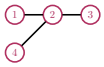
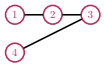
Due: Wednesday, September 25th, 2025
Time: 4:10:5:00pm

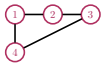
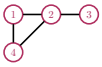
A spanning tree of a connected graph $G = (V, E)$ is a connected subgraph of G whose vertex set is V and has no cycles. A cycle is a (finite) connected subgraph C such that every vertex of C is the endpoint of exactly two edges of C.

For example: if G is the graph: 

The following subgraphs are spanning trees of G:

The following subgraphs are not spanning trees of G :

as they contain a cycle.

1. Let $G = (V, E)$ be a connected graph. Assume $0 <| V |< \infty$ and $E \subset \{\{k, l\} | k, l \in V\}$ be a set of edges for the graph. Let S be the set of all spanning trees of G Assume that we are given and $X_0 \in S$ and X_k for $0 \leq k \leq n$ for some $n \geq 0$. Construct X_{k+1} via the following:

   (i) Randomly (and uniformly) choose an edge $e \in E$ that is not in X_k.

   (ii) Let C denote the unique cycle of $X_k \cup \{e\}$.

   (iii) Choose an edge f uniformly at random from the edges in C.

   (iv) Set $X_{k+1} = X_k \cup \{e\} \setminus \{f\}$.

   You may assume the above defines a Markov chain on S with an appropriate transition matrix $\mathbf{P}$. Using the facts[1] about spanning trees:

   (a) Suppose $i, j \in S$. Assuming that $j = (i \cup \{e\}) \setminus \{f\}$ for some $e, f \in E$ and $e \neq f$, find an expression for p_{ij} and p_{ji}.

   (b) Show that $\{X_k\}_{k \geq 0}$ is an aperiodic, irreducible, Markov chain on S.

   (c) Find the stationary distribution π of $\{X_n\}_{n \geq 0}$.

   (d) Can you describe the importance of the above chain ?

(5 points) for on time submission

[1] Every spanning tree of Ghas exactly $| V | -1$ edges. Every subgraph of G with $| V | -1$ edges and no cycles is a spanning tree. Adding an edge to a spanning tree always creates exactly one cycle.