Due: Thursday October 19th, 2023, 1pm
Problems Due: 1,2

[AWH] Arfken, George B and Weber, Hans J and Harris, Frank E Mathematical methods for physicists: a comprehensive guide *Academic press*, 2011.

1. *(Legendre's Equation)* Let $\lambda \in \mathbb{R}$ and $x : (-1, 1) \to \mathbb{R}$ solve the differential equation

$$(1 - t^2)\frac{d^2x}{dt^2}(t) - 2t\frac{dx}{dt} + \lambda(\lambda + 1)x(t) = 0,$$

for $t \in (-1, 1)$. Assume that x is real analytic and is given by

$$x(t) = \sum_{n=0}^{\infty} a_n t^n,$$

for $t \in (-1, 1)$ and $a_n \in \mathbb{R}$.

   (a) *(Legendre Series)* Find the general solution for any fixed λ.

   (b) *(Legendre Poylnomial)* Suppose $\lambda = m$ for some natural number m then show that there is a particular solution x which is a polynomial.

2. *(Hermite's Equation)* Let $\lambda \in \mathbb{R}, x : \mathbb{R} \to \mathbb{R}$ solve the differential equation

$$\frac{d^2x}{dt^2}(t) - 2t\frac{dx}{dt} + 2\lambda x(t) = 0,$$

for $t \in \mathbb{R}$. Assume that x is real analytic and is given by

$$x(t) = \sum_{n=0}^{\infty} a_n t^n,$$

for $t \in \mathbb{R}$ and $a_n \in \mathbb{R}$.

   (a) *(Hermite Series)* Find the general solution for any fixed λ.

   (b) *(Hermite Poylnomial)* Suppose $\lambda = m$ for some natural number m then show that there is a particular solution x which is a polynomial.

3. Problem 7.5.7 in [AWH]