

24/10/07

Gravitational Radiation

(Similar to the case of EM radiation - moving charge gives rise to EM rad.)

(We take weak field approx. - not unreasonable bcs we want to study grav. field away from the source)

We use weak field approximation:-

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

↪ small

Einstein's eqns :-

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -8\pi G T_{\mu\nu}$$

⇒ [Contracting with $g^{\mu\nu}$]

$$\rightarrow -R = -8\pi G T^\lambda{}_\lambda$$

$$\Rightarrow R_{\mu\nu} = -8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\lambda{}_\lambda \right)$$

(For weak field approx. →

- ① sources shouldn't be strong
- ② ^{sufficiently} far away from any source

Here let us first assume the source is weak & see how far we can go)

Keep terms linear in h on LHS & zeroth order in h on RHS

↪ bcs we already have the energy-momentum tensor

Ex. Check that we get

$$\square h_{\mu\nu} - \frac{\partial^2}{\partial x^\lambda \partial x^\mu} h^\lambda{}_\nu - \frac{\partial^2}{\partial x^\lambda \partial x^\nu} h^\lambda{}_\mu + \frac{\partial^2}{\partial x^\mu \partial x^\nu} h^\lambda{}_\lambda = -8\pi G S_{\mu\nu}$$

\Downarrow
 $T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\lambda{}_\lambda$

All indices are raised & lowered by $\eta_{\mu\nu}$

Define:- $R^{(1)}_{\mu\nu} = \frac{1}{2} \left[\square h_{\mu\nu} - \frac{\partial^2}{\partial x^\alpha \partial x^\mu} h^\alpha{}_\nu - \frac{\partial^2}{\partial x^\alpha \partial x^\nu} h^\alpha{}_\mu + \frac{\partial^2}{\partial x^\mu \partial x^\nu} h^\alpha{}_\alpha \right]$

Now, $T_{\mu\nu} = S_{\mu\nu} - \frac{1}{2} S^\alpha{}_\alpha \eta_{\mu\nu}$

$D_\mu T^{\mu\nu} = 0 \xrightarrow[\text{field limit}]{\text{weak}} \partial_\mu T^{\mu\nu} = 0$

↓
indices raised & lowered by η

(We want to solve) →

$$R^{(1)}_{\mu\nu} = -8\pi G S_{\mu\nu}$$

(But this eqn. also implies) →

$$R^{(1)}_{\mu\nu} - \frac{1}{2} R^{(1)} \eta_{\mu\nu} = -8\pi G T_{\mu\nu}$$

⇓

$\eta^{\mu\nu} R^{(1)}_{\mu\nu}$

↳ $g_{\mu\nu} \rightarrow \eta_{\mu\nu}$

Consistency requires

$$\partial_\mu \left(R^{(1)\mu\nu} - \frac{1}{2} R^{(1)} \eta^{\mu\nu} \right) = 0$$

Ex. Check this using the explicit form of $R^{(1)}_{\mu\nu}$.

