$$e^a{}_\nu \longrightarrow \Lambda^a{}_b \, e^b{}_\nu$$

$$\Rightarrow \quad e \longrightarrow \Lambda e$$

Now $\quad E = e^{-1} \longrightarrow e^{-1}\Lambda^{-1}$

$$= \hat{E}\,\Lambda^{-1}$$

$$\therefore \quad E^\mu{}_a \xrightarrow[\text{Lorentz trs.}]{\text{local}} E^\mu{}_b \left(\Lambda^{-1}_{(y)}\right)^b{}_a$$

Suppose we have a ~~tens~~ vector field $A_\mu(x)$

Define :- $\hat{A}_a = E^\mu{}_a(x)\, A_\mu(x)$

(we will now treat this as the indep. variable instead of $A_\mu(x)$)

We could take $\hat{A}_a$ as indep. variables.

\# $\hat{A}_a$ is scalar under general coordinate trs.

$$\hat{A}_a = E^\mu{}_a(x)\, A_\mu(x) \xrightarrow[\text{Lorentz trs.}]{\text{local}} E^\mu_b(x)\, A_\mu(x) \left(\Lambda^{-1}(x)\right)^b{}_a$$

$$= \hat{A}_b(x) \left(\Lambda^{-1}(x)\right)^b{}_a$$

$\therefore$ $\hat{A}_a$ transforms as a covariant vector under local Lorentz trs.

Given A^μ, we define :-

$$\hat{A}^a = e^a_\mu \, A_\mu$$

$\longrightarrow$ scalar under general coord. trs. & contravariant under local Lorentz trs.

$$\hat{A}^a(y) \longrightarrow \Lambda^a{}_b(y)\,\hat{A}^b(y)$$

In general :-

Given $A^{\mu_1 \ldots \mu_n}{}_{\nu_1 \ldots \nu_m}$

we define :-

$$\hat{A}^{a_1 \ldots a_n}{}_{b_1 \ldots b_m} = e^{a_1}{}_{\mu_1} e^{a_2}{}_{\mu_2} \ldots e^{a_n}{}_{\mu_n} \times E^{\nu_1}{}_{b_1} \ldots E^{\nu_m}{}_{b_m} \times A^{\mu_1 \ldots \mu_n}{}_{\nu_1 \ldots \nu_m}$$

Guaranteed ↓
Whatever new action we get in terms of the vierbein should be inv. under local Lor. trs. bcos the original action was inv. under local trs.
— though this fact isn't manifest.
Possible confusion comes from derivatives
→ So what are the prop. which ~~gives~~ makes local Lor. trs.?

Q) How to define covariant derivatives of tensors of local Lorentz trs.?

$$\mathcal{D}_\mu \hat{A}_b = \underbrace{E^\nu{}_b}\, D_\mu A_\nu$$

$D_\mu A_\nu$ ↳ invariant under local L.T. bcs A_ν has no knowledge about local L.T.

guaranteed to have the right trs. laws but need to work this out

Here there is no fundamental gauge field to compensate for the ∂_μ term — but here we can if introduce indep. gauge fields for local L.T. — that will be too many d.o.f.

So the normal way of defining covariant deriv. doesn't work here

$$= E^\nu{}_b \left(\partial_\mu A_\nu - \Gamma^\rho_{\mu\nu} A_\rho \right)$$