

30/7/07

Generalisation of Maxwell's eqn to the curved space.

Flat space version :-

$$\eta^{\mu\nu} \eta^{\rho\sigma} \partial_\mu F_{\nu\rho} + J^\tau = 0$$

$$\Rightarrow \eta^{\mu\nu} \eta^{\rho\sigma} \partial_\tau \partial_\mu F_{\nu\rho} + \partial_\tau J^\tau = 0$$

||
0 (due to antisym.)

$$\Rightarrow \partial_\tau J^\tau = 0$$

[We ask is the above eqn. satisfied or does it give some additional condn? It would be strange if it gives some " " bcs Maxwell's eqn with source term should contain all info. - let's see what is $\partial_\tau J^\tau$]

$$J^\mu = \sum_n e_n \int du \delta^{(4)}(x^\nu - X_n^\nu(u)) \frac{dX_n^\mu}{du}$$

$$\therefore \partial_\mu J^\mu = \frac{\partial}{\partial x^\mu} J^\mu = \sum_{\mu=0}^3 \sum_n e_n \int du \left(\prod_{\nu \neq \mu} \delta(x^\nu - X_n^\nu(u)) \right) \left\{ \begin{array}{l} \delta'(x^\mu - X_n^\mu(u)) \\ \frac{dX_n^\mu(u)}{du} \end{array} \right.$$

Why, not ∂_μ act on $\frac{dX_n^\mu}{du}$?

→ bcs X_n^μ is just the particle trajectory & the delta fn is the one which tells us that when we happen to be on the trajectory, we will have non-zero source action

$$-\frac{\partial}{\partial x^\mu} \delta(x^\mu - X_n^\mu(u)) = -\frac{d}{du} \delta(x^\mu - X_n^\mu(u))$$

(All sums & products will be written explicitly - here we won't follow summation convention.)

$$= - \sum_n e_n \int du \left(\prod_{\nu \neq \mu} \delta(x^\nu - X_n^\nu(u)) \right) \frac{d}{du} \delta(x^\mu - X_n^\mu(u))$$

$$= - \sum_n e_n \left[\prod_{\nu=0}^3 \delta(x^\nu - X_n^\nu(u)) \right]_{u=-\infty}^{u=\infty}$$

$$= 0$$

Consider $\delta(x^0 - X^0(u))$

$$\text{Now, } X^0(-\infty) = -\infty$$

$$X^0(\infty) = \infty$$

(locus trajectory is there from infinite past to infinite future)

$$\therefore \delta(x^0 - X^0(u)) = 0 \text{ for any finite } x^0$$

This is expected plus

This shows charge is conserved in a covariant way — This extra condition follows automatically even if e.o.m. isn't satisfied

So, we don't get any additional conditions or charges — then that would have implied that the motion of the charges can be det. from this constraint → but this should not be the case; the condition must be satisfied automatically if eqns of motion are satisfied

consistency of the E.O.M of EM

Claim

as it is a classical th. & once we specify initial & final cond., trajectory is fixed

$$\partial_\nu (g^{\mu\nu} g^{\rho\sigma} \partial_\mu F_{\rho\sigma}) = 0$$

$$\partial_\nu J^\nu = 0$$

To prove, go to locally inertial frame.

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad T^\mu_{\nu\rho} = 0 \Rightarrow \partial_\rho g_{\mu\nu} = 0$$

Both follow automatically with imposing additional conditions on the charges or fields

