

\therefore ① reduces to $f_1(x', \vec{F}(x', \vec{x}')) = f_1(x', \vec{x}')$

for $dx'^k dx'^l$:-

$$\textcircled{3} \quad f_{3ij}(x', \vec{F}) \frac{\partial F^i}{\partial x'^k} \frac{\partial F^j}{\partial x'^l} = f_{3kl}(x', \vec{x}')$$

Homogeneity :-

Given any two points: (x, \vec{x}) & (x, \vec{y}) ,
there is an isometry F such that $\vec{F}(x, \vec{x}) = \vec{y}$

This implies $f_1(x', \vec{y}) = f_1(x', \vec{x})$ for any pair of points (\vec{x}, \vec{y}) ~~in space~~ in space
[follows from ①]

$$\Rightarrow \boxed{f_1(x', \vec{x}) = f_1(x')} \quad (\text{indep. of } \vec{x})$$

(Isotropy is not going to give any further restriction on f_1 bcos anyway it is indep. of \vec{x})

$$\# \quad f_{3ij}(x', \vec{F}) \frac{\partial F^i}{\partial x'^k} \frac{\partial F^j}{\partial x'^l} = f_{3kl}(x', \vec{x}')$$

Fix x' (think of it as a parameter without thinking 4D spacetime) & think of f_{3ij} as a 3-dim. metric.

Then, $x^i = F^i(\vec{x}')$ is an isometry of the 3-dimensional metric $f_{3ij}(x', \vec{x}')$ for every x' .

Isometries associated to the space we consider here are homogeneity & isometry

$$ds^2 = f_{3ij}(t, \vec{x}) dx^i dx^j$$

& make a coord. trs. $x^i = F^i(\vec{x}')$
 then this is an isometry of the 3-d metric

$$\text{if } \left[f_{ij}(t, \vec{x}) \frac{\partial F^i}{\partial x'^k} \frac{\partial F^j}{\partial x'^l} = f_{3kl}(t, \vec{x}') \right]$$

is satisfied.

the statement ^{about} the 4-d metric is converted to the statement about the 3-d metric — the coord. can be restated in the 3-d language without any reference to time.

→ Reinterpretation

This ↓ could be done bcos we have the notion of COSMIC TIME
 → any isometry trs. should never change that time

List of homogeneous and isotropic metrics in $D=3$

① flat space :-

$$f_{3ij} dx^i dx^j$$

$$= \hat{f}(t) \left\{ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right\}$$

↓
arbitrary fn of time

$\vec{x}' \rightarrow \vec{x}' + \vec{a}$: establishes homogeneity
 $SO(3)$ rotation around origin → ~~isometry~~
 establishes isotropy

② Surface of a 3-dimensional sphere S^3

$$\sum_{i=1}^4 (z^i)^2 = R^2$$

$$ds^2 = 2z^i dz^i$$

$SO(4)$

Ricci scalar is -ve & const. → +ve curvature metric

Given any 2 pts. on the 2D sphere S^2 , there is a rot. about origin which takes first pt. to the other pt.

