27/7/07 [We derived
Eqns of motion of a charged particle in presence of a background metric & a background EM
– Now we address the reverse problem...]

Generalise Maxwell's eqns in presence of gravity :–

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \rightarrow \text{Bianchi identities}$$

These are consequences of $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

In the component notation, they corr. to

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \& \quad \vec{\nabla}\times\vec{E} + \partial\vec{B}/\partial t = 0$$

$E_i = -F_{0i}$ where $i = 1,2,3$

$F_{ij} = \epsilon_{ijk} B_k$ where $i,j,k = 1,2,3$

In absence of gravity

$$\eta^{\mu\nu}\partial_\mu F_{\nu\sigma}(x) = -\eta_{\sigma k} J^k(x)$$

where $J^0(x) = \rho(\vec{x},t)$ → charge density

& $J^i = j^i(\vec{x},t)$ → current

These eqns are used to solve for the vector potential

In the presence of gravity, (Bianchi identities generalise to) →

$$D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0$$

Ex: Show that this is equivalent to:

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$$

[Γ's cancel]

even though it doesn't explicitly look covariant, it is indeed covariant

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Also, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = D_\mu A_\nu - D_\nu A_\mu$ → Γ's cancel

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[We have to know the trs. prop. of $J^\mu(x)$, which isn't an indep. quantity, to get a cov. eqn from $\eta^{\mu\nu}\partial_\mu F_{\nu\sigma} = -\eta_{\sigma\kappa} J^\kappa$]

~~What is known~~

What is J^μ in flat space-time?

→ Suppose we have several particles with charges $e_1, e_2, \ldots$ with trajectories $\vec{x}_1(t), \vec{x}_2(t), \ldots\ldots$

The charge density is given by

$$\rho(\vec{x}, t) = \sum_n e_n\, \delta^{(3)}(\vec{x} - \vec{x}_{(n)}(t))$$

$$\therefore\ J^i(\vec{x}, t) = \sum_n e_n \frac{d\vec{x}^{\,i}_{(n)}}{dt}\, \delta^{(3)}(\vec{x} - \vec{x}_{(n)}(t))$$

Manifestly Lorentz inv. form

Represent the trajectory as

$$x^\mu = X^\mu(u)$$

u → parameter labelling the trajectory

e.g. → if $u = x^0$, $x^0 = u^0$ → first eqn