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Spontaneous symmetry breaking

Suppose we have a continuous symmetry group G .

Suppose we have a set of scalar fields

$$\vec{\Phi} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix} \text{ is a representation } R \text{ of } G$$

$$\text{For } g \in G, \vec{\Phi} \rightarrow R(g) \vec{\Phi}$$

↳ representation matrix

ϕ can be taken to be real or complex
↓
take twice...

the reps. need not be irreducible

Suppose $V(\vec{\Phi})$ is the potential

$$V(R(g)\vec{\Phi}) = V(\vec{\Phi}) \text{ since } G \text{ is the symmetry group of } V(\vec{\Phi}).$$

Suppose $V(\vec{\Phi})$ is a minimum at $\vec{\Phi}_0$.

Then $R(g)\vec{\Phi}_0$ is also a minimum of $V(\vec{\Phi})$.

In general $R(g)\vec{\Phi}_0 \neq \vec{\Phi}_0$

$\# \mathcal{H} \subset G$: collection of elements of G such that $R(h)\vec{\Phi}_0 = \vec{\Phi}_0$ if and only if $h \in \mathcal{H}$.

look at a subset of elements which leave $\vec{\Phi}_0$ unchanged

(1) $I \in \mathcal{H}$

(2) If $h_1, h_2 \in \mathcal{H}$

$$\Rightarrow R(h_1)\vec{\Phi}_0 = \vec{\Phi}_0, R(h_2)\vec{\Phi}_0 = \vec{\Phi}_0$$

$$\Rightarrow R(h_1 h_2)\vec{\Phi}_0 = R(h_1)R(h_2)\vec{\Phi}_0$$

$$\Rightarrow R(h_1) R(h_2) \vec{\Phi}_0 = R(h_1) \vec{\Phi}_0 = \vec{\Phi}_0$$

$$\Rightarrow h_1, h_2 \in \mathcal{H}$$

$$3) R(h) \vec{\Phi}_0 = \vec{\Phi}_0 \Rightarrow \vec{\Phi}_0 = R(h)^{-1} \vec{\Phi}_0 = R(h^{-1}) \vec{\Phi}_0$$

$$\Rightarrow h^{-1} \in \mathcal{H} \text{ if } h \in \mathcal{H}$$

these
 \Rightarrow
 prop.
 show
 that

\mathcal{H} is a subgroup of G

Infinitesimal transformations

$$\mathfrak{g} = \mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a T^a \quad [T^1, \dots, T^{n_G} \text{ are generators of } G]$$

We take T^1, \dots, T^{n_G} to be generators of \mathcal{H} .

$$\therefore h = \mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a T^a$$

what linear comb. of generators we choose is upto us

$$V \left(R \left(\mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a T^a \right) \vec{\Phi} \right) = V(\vec{\Phi})$$

$$\mathbb{1} + i \sum_{a=1}^{n_G} \epsilon^a R(T^a)$$

$$\Rightarrow V \left(\vec{\Phi} + i \sum_{a=1}^{n_G} \epsilon^a R(T^a) \vec{\Phi} \right) = V(\vec{\Phi})$$

$$\Rightarrow V \left(\vec{\Phi}_0 + i \sum_{a=1}^{n_G} \epsilon^a R(T^a) \vec{\Phi}_0 \right) = V(\vec{\Phi}_0)$$

min. of pot. has certain flat dirns along these derivations

