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Consider a set of variables u_1, u_2, \dots, u_n , which are the integration variables.

We want to calculate

$$\langle O \rangle = \frac{\int \left(\prod_i du_i \right) e^{iS(\vec{u})} O(\vec{u})}{\int \left(\prod_i du_i \right) e^{iS(\vec{u})}}$$

This in principle, can be a prod. of several fields

$n = 4NK$ for pure gauge theory

on 3+1 dim. spacetime

dim. of G
of lattice points

Symmetries :-



$$u_i \xrightarrow[\text{transform}]{\text{gauge}} F_i(\vec{u}, \vec{\theta})$$

|
gauge
trs. parameters

set of sym. trans. labelled by θ_α 's

At each lattice pt. we will have K gauge trs. parameters - so total = NK

$$\theta_\alpha : \alpha = 1, 2, \dots, NK$$

G :- group of symmetries generated by $\vec{\theta}$

dimension NK

$G \rightarrow$ gauge group

$$\mathcal{G} = G \otimes G \otimes \dots \otimes G \quad (N \text{ times})$$

$S(\vec{u}) = S(F(\vec{u}, \vec{\theta})) \rightsquigarrow$ action is invariant

$O(\vec{u}) = O(F(\vec{u}, \vec{\theta})) \rightsquigarrow$ O is a gauge invariant operator

assumption of g

$$u_i = F_i(\vec{u}, \vec{\theta}), \text{ then } \prod_{i=1}^n du_i = \prod_{i=1}^n du_i$$

$$\text{i.e., } \det \left(\frac{\partial F_i(\vec{u}, \vec{\theta})}{\partial u_j} \right) = 1$$

If these are not satisfied, then there is no hope to get gauge-inv. result bcs the ^{integration} measure on change of variables shouldn't change, in addn to $O(\vec{u}) = O(F(\vec{u}, \vec{0}))$

Ex. Check that $\prod_{\mu=0}^3 \prod_{a=1}^K \mathcal{D}A_{\mu}^a$ is gauge invariant.

(i.e., check that the int. measure doesn't change under gauge trs. in the above case)

[We can separately take the π det. & the integral to be gauge inv. but the 2 together do not. But to make things simpler, the above method is better. Thus, the whole analysis will be carried out assuming that there is a symmetry grp. $\mathfrak{g} = \mathfrak{g} \oplus \mathfrak{h} \oplus \dots \oplus \mathfrak{g}$ - it's another matter that $\mathfrak{g} = \mathfrak{g} \oplus \mathfrak{h} \oplus \dots \oplus \mathfrak{g}$]

$$\langle O \rangle = \frac{\int \left(\prod_i d\mu_i \right) e^{iS(\vec{u})} O(\vec{u})}{\int \left(\prod_i d\mu_i \right) e^{iS(\vec{u})}} = \frac{N}{Z}$$

not necessary the direct product matrix repr.

Suppose we have some representation of \mathfrak{g}

Generators of \mathfrak{g} are T^A [$A=1, 2, \dots, NK$]

Now,

$$U(\vec{\theta} + \delta\vec{\theta}) U(\vec{\theta})^{-1} = \mathbb{1} + i T^A \delta_{\omega} \theta^A \rightarrow \text{define } \delta_{\omega} \theta^A$$

This relation is indep. of the rep. (or parametrisation) we choose - the relation bet. $\delta_{\omega} \theta^A$ & $\delta\vec{\theta}$ will of course depend on the choice of repr.

A physical field can't be rep. by diff. rep. at diff. pts. - but mathematically, there is no reason why i_1, i_2, \dots, i_{NK} will belong to the same rep.



