How is ϕ^4 theory renormalizable?

According to the given criteria we must add ϕ^3 vertex.

$$\int g^{(3)} \phi^3 \, d^4x$$

$$[g^{(3)}] = 1$$

$$S = \int \left(-\frac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - g/_{4!}\,\phi^4\right) d^4x$$

has a symmetry under $\phi \to -\phi$

action doesn't change

$\Rightarrow$ All n-point fn vanishes identically for odd n.

$\to$ there is no 3-pt. fn. $\Downarrow$ no diag to contribute to it

3-pt. fn is identically zero

$\xRightarrow{\text{hence}}$ we do not need a 3-pt. vertex to cancel divergence.

(Modification) $\to$

If there is an underlying symmetry, then it is enough if we add all terms in the action which are invariant under that symmetry and whose coefficients have dimension ≥ 0.

terms ~~inv~~ not inv. under this sym. aren't generated

For this to work, the regularized theory must also have that symmetry.

e.g. $\to$ $\phi \to -\phi$ symmetry holds in $(4-\epsilon)$ dimension also.

(If regularization breaks this sym, you need the counterterm)

(Most of the reqd. sym. are preserved by dim. reg.)

$$\left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle = \underbrace{(-1)^N}_{\text{must be } 1} \left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle$$

$$\int [D\phi]\, e^{iS[\phi]} \prod_i \phi(x_i) \Big/ \int [D\phi]\, e^{iS[\phi]}$$

Now $\chi = -\phi$

$$= \frac{\int [D\chi]\, \underbrace{e^{iS[-\chi]}}_{= e^{iS[\chi]}} \prod_{i=1}^{N} \chi(x_i)\, (-1)^N}{\int [D\chi]\, \underbrace{e^{iS[-\chi]}}_{= e^{iS[\chi]}}}$$

$$= (-1)^N \left\langle \prod_{i=1}^{N} \phi(x_i) \right\rangle$$

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in $D=2$ dim.

$$\frac{\partial}{\partial k^2} \int \frac{(dl)^D}{(l)^{\alpha-2}}$$

$$\underbrace{A}_{\text{div.}} + \underbrace{Bk^2}_{\text{div.}} + \text{finite}$$

we must have $(n+2)\left(\frac{2-D}{2}\right) + D - 2 \neq 0$ ... coupling const.

$$(n+2)\left(\frac{2-D}{2}\right) + D = 2$$

$(n+2)$ pt. fn. has max. deg. of divergence

$$= (n+2)\,\frac{2-D}{2} + D = 2$$

For $\phi^n \partial_\mu \phi \partial^\mu \phi$, to cancel the div. for