

12/1/06

Huang → Textbook

# Statistical Mechanics (Classical)

→ deal with a classical dynamical system (Hamiltonian) with large no. of degrees of freedom.

↓ <sup>in general</sup> will be described by

$$(q^1, \dots, q^N, p_1, \dots, p_N)$$

Example :- Gas of  $M$ -particles  $\Rightarrow N = 3M$

(will be interested in the case)  $N$ : large no.  $\sim 10^{23}$

In practice, it is impossible to solve the dynamics.

Initial conditions are unknown.

Statistical Mechanics  $\rightarrow$  A way to extract certain average properties of such a system.

↳ Requires certain postulates which we shall not prove.

Consider a dynamical system with large no. of degrees of freedom & let it evolve in time starting from a given ~~instant~~ initial condition.

Postulate :- After certain time, the system reaches an equilibrium state in which certain quantities  $f(\vec{q}, \vec{p})$  do not change with time.  $\rightarrow$  MACROSCOPIC QUANTITIES

Even if the system has symmetry, it is impossible to know initial cond.  
 $\rightarrow$  e.g. non-int. gas molecules in a box

In principle one should be able to prove them as we have a classical dyn. system

Approximate statement (because)

(1)  $F$  can change by small amount

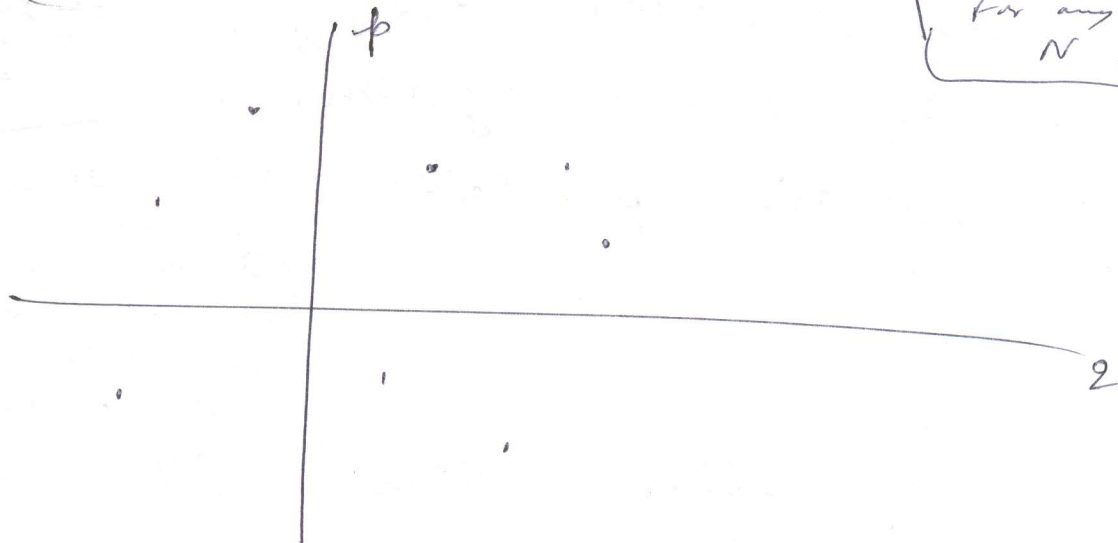
— (this can happen) frequently

(2)  $F$  can change by large amount — rarely

In equilibrium,  $F$  can be calculated by time average of  $F$ .

$$F = \langle F \rangle = \frac{1}{n} \left[ F(\bar{q}(t_1), \bar{p}(t_1)) + F(\bar{q}(t_2), \bar{p}(t_2)) + \dots + F(\bar{q}(t_n), \bar{p}(t_n)) \right]$$

(these are  $n$  pts. in the phase space)



( $\langle F \rangle$  is the ~~time~~ avg. of values of  $F$  at these pts.)

$\langle F \rangle \rightarrow$  can be regarded as an average over multiple systems at the same time but different initial conditions.

$\rightarrow$  Statistical ensemble.

It's not that all quantities are not changing — as it is a dynamical sys.

Some quantities, like  $\bar{q}$ ,  $\bar{p}$  do change with time

If we take  $N$  to be large,  $F$  is almost constant

$F$  is const. becomes true only in the  $N \rightarrow \infty$  limit

For any finite  $N$ ,  $F$  is not a true const. of motion — but it behaves as a const. of motion most times

$N = \infty$  isn't a dyn. system, for any dyn. sys.,  $N$  is finite

